

Optimization of heat production for electricity market participation

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Abstract. In this chapter, we introduce optimization methods for production scheduling and power market participation for district heating systems under uncertainty. We present an optimization model for scheduling the production units using mixed-integer linear programming and stochastic programming. Based on the optimal production scheduling, the bidding amounts and prices to the day-ahead market are determined using an extension of the model. Results are shown for a demo case of a Danish district heating system.

Keywords: Optimal production planning · Sector Coupling · Renewable Energy · Electricity Markets · Bidding.

1 Introduction

We focus on the operational production planning of the district heating (DH) operator. Usually, the operator has a set of different heat production units at their disposal to fulfill the heat demand in the network. Those units can be heat production units converting an input energy carrier to only heat, or combined heat and power (CHP) units that produce heat and power simultaneously. The input energy carriers can be, for example, natural gas, electricity or wood chips. Further sources of heat available to the DH operator can be solar thermal units or industrial waste heat. With an increasing number of small-scale production units, changes in heat demand, volatility in electricity prices and heat production from renewable energy sources (RES) as well as the availability of storage possibilities, the production planning becomes a complex optimization problem for the DH operator. This is in particular the case for low-temperature district heating systems (DHS), since there, the inclusion of heat pumps as well as waste heat and solar thermal production is widely used [14, 12]. Furthermore, the production moves from a few central units to many smaller central and decentralized units [12]. Therefore, a large number of units need to be dispatched while

many of them are coupled to the electricity markets. Additionally, production of RES units is uncertain at the time of planning.

In the remainder of this chapter, we first present an optimization model to dispatch the units under uncertain conditions. Secondly, we will focus on how the production optimization can be used to determine electricity market bids of market-dependent units. The planning problem is formulated using the time scale relevant to the market. For example, the day-ahead electricity market Elspot of the Nordic electricity market has hourly planning intervals [19]. Here, the hourly dispatch of the units is of interest for the DH provider. The models and methods presented in this chapter are applicable to any length of market periods. The bidding method explained in Section 3 relates to a deregulated wholesale electricity market where the market is cleared and the price is determined based on the intersection between supply and consumption bids. For instance, all bids to Elspot have to be submitted before noon the day before energy delivery. Then the market operator calculates the market clearing price for each hour individually by sorting all production bids in ascending order of price and the consumption bids in descending order of price. The market clearing price is the price at which both curves meet [19]. In other market settings such as regulated electricity systems, the bidding method may be obsolete. However, the dispatch model presented in Section 2 based on electricity prices is still relevant. In Section 4, we present results of the optimization for a representative week in October 2020 in the Danish DHS in Brønderslev.

2 Mathematical Optimization for Operational Production Planning

The operational production optimization or economic dispatch of units in a DHS can be formulated and solved using a mixed-integer linear program (MILP) as several publications show. For example, [5] and [22] formulate different MILPs to schedule CHP production units in connection with boilers, thermal storage tanks and solar heat units. The models are used for cost evaluations, therefore, the authors investigate predetermined price and solar heat scenarios using a deterministic model. [13] optimize an integrated power and heating system including a wind farm, a heat pump that can be produce based on the wind power production and a CHP unit. They model this particular system including the connected DH and power systems.

To appropriately consider uncertain prices and production in the operational planning, we move from a deterministic model to stochastic programming. For example, [15] optimize the operation of CHP units in combination with electric boilers and heat pumps under heat demand and electricity price uncertainty. An overview of models and optimization methods for DH is given in [20].

Here, we also formulate the dispatch problem as a two-stage stochastic program [1] using a MILP. It is considered a stochastic program since at the time of optimization, electricity prices as well as heat production from solar heat and waste heat are still unknown. Heat demand is considered deterministic one day

in advance, since it can be predicted quite accurately by the methods shown in Chapter 6. The mathematical formulation that is presented in this section is adapted from the model formulations described in [3] and [21]. An overview of the nomenclature is given Table 1.

We are presenting the dispatch optimization problem for a typical DH system which contains several types of heat production units, given by the set \mathcal{U} . Some units are dependent on the electricity markets, because they are either CHP units \mathcal{U}^{CHP} producing electricity or operated with electricity \mathcal{U}^{EL} , e.g., heat pumps or electric boilers. There could be other units \mathcal{U}^{N} for which the production is determined by external factors and are therefore non-dispatchable, e.g., solar thermal units or waste heat injection. The remaining units \mathcal{U}^{H} are heat production units independent of the electricity market and can be freely scheduled within their capacities, e.g., biomass or gas boilers. All types of units are taken into account when fulfilling heat demand. In this planning problem, the network is abstracted by considering several demand sites \mathcal{D} with given heat demand for each time period \mathcal{T} . Thus, the demand is represented in terms of energy where supply temperatures are taken into account when calculating the heat demand. We distinguish several demand sites to model features of the network topology, e.g., that not all units can reach all demand sites. The goal is to fulfill the heat demand at all demand sites in all scenarios Ω of electricity prices and heat inflow from non-dispatchable units. Additionally, the DH operator can use heat storage tanks \mathcal{S} to store heat from one period to the next.

The decisions that have to be taken are:

- heat production $q_{u,t,\omega}$ of each unit $u \in \mathcal{U}$ in each period $t \in \mathcal{T}$ and scenario $\omega \in \Omega$ and how this production is distributed to the demand sites ($x_{u,d,t,\omega}^{\text{UD}}$) and storage tanks ($x_{u,s,t,\omega}^{\text{US}}$).
- storage levels $\delta_{s,t,\omega}$ of each storage tank $s \in \mathcal{S}$ in each period $t \in \mathcal{T}$ and scenario $\omega \in \Omega$ and the usage of the storage for fulfilling the demand ($x_{d,s,t,\omega}^{\text{DS}}$).
- commitment decisions for units with minimum up- and down times that are given by the set $\mathcal{U}^{\text{C}} \subseteq \mathcal{U}$. Commitment decisions are the on/off status ($y_{u,t,\omega}$) as well as the periods where a unit is started ($y_{u,t,\omega}^{\text{Start}}$) or stopped ($y_{u,t,\omega}^{\text{Stop}}$).

The objective function (equation (1)) minimizes the expected operational cost of heat production taking heat production (C_u^{H}) and start-up costs (C_u^{S}) as well electricity purchases of electricity-driven units and electricity sales of CHP units into account. The electricity prices ($\lambda_{t,\omega}$) are considered uncertain and the power production/consumption is calculated based on the heat production and the heat-to-power ratio ϕ_u . The costs are weighted with the probability (π_ω) of the respective scenario.

$$\begin{aligned} \text{Min} \sum_{\omega \in \Omega} \pi_\omega \sum_{t \in \mathcal{T}} \left[\sum_{u \in \mathcal{U}} C_u^{\text{H}} q_{u,t,\omega} + \sum_{u \in \mathcal{U}^{\text{C}}} C_u^{\text{S}} y_{u,t,\omega}^{\text{Start}} \right. \\ \left. + \sum_{u \in \mathcal{U}^{\text{EL}}} \lambda_{t,\omega} \phi_u q_{u,t,\omega} - \sum_{u \in \mathcal{U}^{\text{CHP}}} \lambda_{t,\omega} \frac{q_{u,t,\omega}}{\phi_u} \right] \quad (1) \end{aligned}$$

Table 1: Nomenclature

Sets	
$\mathcal{T} = \{1, \dots, \mathcal{T} \}$	Set of time periods t
\mathcal{U}	Set of heat production units u , $\mathcal{U} = \mathcal{U}^{\text{CHP}} \cup \mathcal{U}^{\text{EL}} \cup \mathcal{U}^{\text{H}} \cup \mathcal{U}^{\text{N}}$
$\mathcal{U}^{\text{CHP}} \subset \mathcal{U}$	Subset of CHP production units
$\mathcal{U}^{\text{EL}} \subset \mathcal{U}$	Subset of electricity-based production units
$\mathcal{U}^{\text{H}} \subset \mathcal{U}$	Subset of heat-only production units
$\mathcal{U}^{\text{N}} \subset \mathcal{U}$	Subset of non-dispatchable production units
$\mathcal{U}^{\text{C}} \subset \mathcal{U}$	Subset of production units needing commitment decisions
\mathcal{D}	Subset of demand sites
\mathcal{S}	Set of heat storage tanks s
Ω	Set of scenarios ω
\mathcal{B}	Set of block bids b
Parameters	
C_u^{H}	Cost for producing heat with unit $u \in \mathcal{U}$ [EUR/MWh]
C_u^{S}	Start-up cost of unit $u \in \mathcal{U}^{\text{CHP}}$ [EUR]
$\underline{Q}_u, \overline{Q}_u$	Minimum and maximum heat production for unit $u \in \mathcal{U}$ [MW]
$\Delta_{u,d}^{\text{UT}}, \Delta_{u,d}^{\text{DT}}$	Minimum up- and down-time of unit $u \in \mathcal{U}^{\text{CHP}}$ [time periods]
$A_{u,d}^{\text{UD}}$	Binary parameter: 1, if unit $u \in \mathcal{U}$ is connected to demand site $d \in \mathcal{D}$, 0 otherwise
$A_{u,s}^{\text{US}}$	Binary parameter: 1, if unit $u \in \mathcal{U}$ is connected to the thermal storage s , 0 otherwise
$A_{s,d}^{\text{SD}}$	Binary parameter: 1, if storage tank $s \in \mathcal{S}$ is connected to demand site $d \in \mathcal{D}$, 0 otherwise
φ_u	Heat-to-power ratio for unit $u \in \mathcal{U}^{\text{CHP}}$ [MWh/MWhe]
$S_s^{\text{I}}, S_s^{\text{T}}$	Initial and target storage level in storage s [MWh]
K_s	Capacity of storage s [MWh]
l_s	Losses per period in storage s (%)
$\lambda_{t,\omega}$	Electricity price for time period $t \in \mathcal{T}$ and scenario $\omega \in \Omega$ [EUR/MWhe]
$B_{d,t}$	Heat demand at site $d \in \mathcal{D}$ in time period $t \in \mathcal{T}$ [MWh]
$I_{u,t,\omega}$	Stochastic heat input from heat production unit $u \in \mathcal{U}^{\text{N}}$ in period $t \in \mathcal{T}$ and scenario $\omega \in \Omega$ [MWh]
π_ω	Probability of scenario $\omega \in \Omega$
H	Time-factor, fraction of one hour
$v_{b,\omega}$	value of bid $b \in \mathcal{B}$ in scenario $\omega \in \Omega$
a_b	bidding quantity of bid $b \in \mathcal{B}$
Variables	
$y_{u,t,\omega} \in \{0, 1\}$	Binary variable, status of unit $u \in \mathcal{U}^{\text{CHP}}$ and period $t \in \mathcal{T}$, 1 = on, 0 = off
$y_{u,t,\omega}^{\text{Start}} \in \{0, 1\}$	Binary variable, 1 if unit $u \in \mathcal{U}^{\text{CHP}}$ is started period $t \in \mathcal{T}$, 0 otherwise
$y_{u,t,\omega}^{\text{Stop}} \in \{0, 1\}$	Binary variable, 1 if unit $u \in \mathcal{U}^{\text{CHP}}$ is stopped period $t \in \mathcal{T}$, 0 otherwise
$q_{u,t,\omega} \in \mathbb{R}^+$	Heat production of unit $u \in \mathcal{U}$ in period $t \in \mathcal{T}$ and scenario $\omega \in \Omega$ [MWh]
$x_{u,d,t,\omega}^{\text{UD}} \in \mathbb{R}^+$	Heat flow from unit $u \in \mathcal{U}$ to demand site $d \in \mathcal{D}$ in period $t \in \mathcal{T}$ and scenario $\omega \in \Omega$ [MWh]
$x_{u,s,t,\omega}^{\text{US}} \in \mathbb{R}^+$	Heat flow from unit $u \in \mathcal{U}$ to storage $s \in \mathcal{S}$ in period $t \in \mathcal{T}$ and scenario $\omega \in \Omega$ [MWh]
$x_{s,d,t,\omega}^{\text{SD}} \in \mathbb{R}^+$	Heat flow from storage $s \in \mathcal{S}$ to demand site $d \in \mathcal{D}$ in period $t \in \mathcal{T}$ and scenario $\omega \in \Omega$ [MWh]
$\sigma_{s,t,\omega} \in \mathbb{R}^+$	Storage level of storage $s \in \mathcal{S}$ in period $t \in \mathcal{T}$ and scenario $\omega \in \Omega$ [MWh]
$z_b \in \{0, 1\}$	Binary variable, 1 if bid b is selected, 0 otherwise

Constraint (2) ensures that the heat demand ($B_{d,t}$) of each demand site is fulfilled by heat flow either directly from the production units or the storage tanks. Constraint (3) distributes the production from each unit either to the demand sites or the storage tanks. Heat flow between components that are not possible are excluded by constraints (4) to (6) by using the binary parameters $A_{u,d}^{\text{UD}}$, $A_{u,s}^{\text{US}}$ and $A_{s,d}^{\text{SD}}$ (being 1, if there is a connection, 0, otherwise) and the maximum capacities of units \bar{Q}_u and storage tanks K_s .

$$\sum_{u \in \mathcal{U}} x_{u,d,t,\omega}^{\text{UD}} + \sum_{s \in \mathcal{S}} x_{s,d,t,\omega}^{\text{SD}} = B_{d,t} \quad \forall d \in \mathcal{D}, t \in \mathcal{T}, \omega \in \Omega \quad (2)$$

$$q_{u,t,\omega} = \sum_{d \in \mathcal{D}} x_{u,d,t,\omega}^{\text{UD}} + \sum_{s \in \mathcal{S}} x_{u,s,t,\omega}^{\text{US}} \quad \forall u \in \mathcal{U}, t \in \mathcal{T}, \omega \in \Omega \quad (3)$$

$$H x_{u,d,t,\omega}^{\text{UD}} \leq A_{u,d}^{\text{UD}} \bar{Q}_u \quad \forall u \in \mathcal{U}, d \in \mathcal{D}, t \in \mathcal{T}, \omega \in \Omega \quad (4)$$

$$H x_{u,s,t,\omega}^{\text{US}} \leq A_{u,s}^{\text{US}} \bar{Q}_u \quad \forall u \in \mathcal{U}, s \in \mathcal{S}, t \in \mathcal{T}, \omega \in \Omega \quad (5)$$

$$x_{s,d,t,\omega}^{\text{SD}} \leq A_{s,d}^{\text{SD}} K_s \quad \forall s \in \mathcal{S}, d \in \mathcal{D}, t \in \mathcal{T}, \omega \in \Omega \quad (6)$$

The production of the units is limited to its maximum capacity (\bar{Q}_u) in constraints (7). For CHP units, the production is further restricted by the unit's online status (constraint (8)). The online status $y_{u,t,\omega}$ is equal to 1 if unit u is running in time step t and zero otherwise. Correspondingly, the startup (shutdown) status $y_{u,t,\omega}^{\text{Start}}$ ($y_{u,t,\omega}^{\text{Stop}}$) indicates whether unit u is started up (shut down) in time step t . This is set in constraint (9), while constraint (10) disallows simultaneous startup and shutdown of the same unit. Constraints (11) and (12) ensure the minimum up- and down-time (Δ_u^{UT} and Δ_u^{DT}) requirements, respectively. Finally, the production of non-dispatchable units is limited by the production ($I_{u,t,\omega}$) in the given scenario (constraint (13)).

$$H q_{u,t,\omega} \leq \bar{Q}_u \quad \forall u \in \mathcal{U} \setminus \mathcal{U}^{\text{CHP}}, t \in \mathcal{T}, \omega \in \Omega \quad (7)$$

$$\underline{Q}_u y_{u,t,\omega} \leq H q_{u,t,\omega} \leq \bar{Q}_u y_{u,t,\omega} \quad \forall u \in \mathcal{U}^{\text{CHP}}, t \in \mathcal{T}, \omega \in \Omega \quad (8)$$

$$y_{u,t,\omega} - y_{u,t-1,\omega} = y_{u,t,\omega}^{\text{Start}} - y_{u,t,\omega}^{\text{Stop}} \quad \forall u \in \mathcal{U}^{\text{CHP}}, t \in \mathcal{T} \setminus \{1\} \quad (9)$$

$$y_{u,t,\omega}^{\text{Start}} + y_{u,t,\omega}^{\text{Stop}} \leq 1 \quad \forall u \in \mathcal{U}^{\text{CHP}}, t \in \mathcal{T} \quad (10)$$

$$\sum_{\tau=t+1-\Delta_u^{\text{UT}}}^t y_{u,\tau,\omega}^{\text{Start}} \leq y_{u,t,\omega} \quad \forall u \in \mathcal{U}^{\text{CHP}}, t \in \mathcal{T} \quad (11)$$

$$\sum_{\tau=t+1-\Delta_u^{\text{DT}}}^t y_{u,\tau,\omega}^{\text{Stop}} \leq (1 - y_{u,t,\omega}) \quad \forall u \in \mathcal{U}^{\text{CHP}}, t \in \mathcal{T} \quad (12)$$

$$q_{u,t,\omega} \leq I_{u,t,\omega} \quad \forall u \in \mathcal{U}^{\text{N}}, t \in \mathcal{T}, \omega \in \Omega \quad (13)$$

The storage level is updated according to in- and outflow and losses (l_s) in each period in constraints (14) and (15) depending on whether it is the initial period or not. The storage level must be below the capacity (K_s) in all periods (constraint (16)) and above a certain target (S_s^{T}) at the end of the planning horizon (17).

$$\delta_{s,1,\omega} = l_s S_s^I + \sum_{u \in \mathcal{U}} x_{u,s,1,\omega}^{\text{US}} - \sum_{d \in \mathcal{D}} x_{s,d,1,\omega}^{\text{SD}} \quad \forall s \in \mathcal{S}, \omega \in \Omega \quad (14)$$

$$\delta_{s,t,\omega} = l_s \delta_{s,t-1,\omega} + \sum_{u \in \mathcal{U}} x_{u,s,t,\omega}^{\text{US}} - \sum_{d \in \mathcal{D}} x_{s,d,t,\omega}^{\text{SD}} \quad \forall s \in \mathcal{S}, t \in \mathcal{T} \setminus \{1\}, \omega \in \Omega \quad (15)$$

$$\delta_{s,t,\omega} \leq K_s \quad \forall s \in \mathcal{S}, t \in \mathcal{T}, \omega \in \Omega \quad (16)$$

$$\delta_{s,|\mathcal{T}|,\omega} \geq S_s^T \quad \forall s \in \mathcal{S}, \omega \in \Omega \quad (17)$$

Finally, production of the units connected to the electricity market needs to be decided now while the prices and renewable production is still uncertain. Therefore, the decisions on production and commitment for CHP units and electricity-based units needs to be equal in each scenario. This is ensured by constraints (18) and (19).

$$q_{u,t,\omega} = q_{u,t,\omega'} \quad \forall \omega, \omega' \in \Omega, u \in \mathcal{U}^{\text{CHP}} \cup \mathcal{U}^{\text{EL}}, t \in \mathcal{T} \quad (18)$$

$$y_{u,t,\omega} = y_{u,t,\omega'} \quad \forall \omega, \omega' \in \Omega, u \in \mathcal{U}^{\text{C}} \cap (\mathcal{U}^{\text{CHP}} \cup \mathcal{U}^{\text{EL}}), t \in \mathcal{T} \quad (19)$$

Please note that it can be reasonable to extend the model's time horizon beyond the actual planning horizon of interest in order to take storage behaviour over several days into account. [8] and [2] identified a time horizon of five and seven days as most fitting for their respective applications. The model can then be applied in a rolling horizon manner. For example, if we are interested in planning the first day in advance, we can consider seven days to account for storage behaviour. When we plan for the second day, we move the entire time window by one day and consider days two to eight in the model.

The model formulation can both be used for operational production planning and as a basis for optimizing bids to electricity markets such as the day-ahead market, as we further discuss in Section 3.

3 Bidding to Electricity Markets

DH providers have the possibility to further reduce their operational costs by participating in the electricity markets. They can sell electricity production from CHP units in hours with high electricity prices or they can produce heat from electricity, e.g., through electric boilers or heat pumps, in hours with low electricity prices. To participate in the market, the DH provider has to determine bids that are defined by bidding price (EUR/MWhe), bidding amount (MWhe) and valid hour(s).

In general, the DH provider can use bidding methods proposed for bidding of thermal power producers and CHP units to determine the bidding amount for CHP units. For example, [6] and [18] base their bidding prices and amounts on the electricity price forecasts for the next day by solving MILPs containing the price information at certain confidence intervals. The models return the bidding amount at the respective prices. [4] and [11] assume a pre-defined set of bidding

prices and use a stochastic program to determine the bidding amount per unit at each price. [7] use price scenarios to determine the bidding amount for each scenario. The prices and amounts are used to create bidding curves. A similar concept is used in [16], where the authors group several power production units in a Virtual Power Plant (VPP) and bid for them to the day-ahead market and balancing markets based on price scenarios. This concept is adapted for a DH application in [3]. In the context of DH, [17] and [2] use the difference in operational production costs between heat-only units and CHP units to determine the bidding price. This is also the concept that we will present here.

The above-mentioned literature focuses on hourly bids to the markets. However, in many markets it is also possible to group several hours together and bid them jointly to the market in so-called block bids, i.e., all hours have the same price and amount and can only be accepted or rejected in total. [10] and [9] create blocks bid for hydro power units by creating all possible hour blocks and determine the production amounts by optimizing the amount for different price scenarios in a stochastic program. [21] use a similar technique for DH by using the price difference in heat production units as basis for bidding. This concept will also be presented in the remainder of this section.

3.1 Unit-switching prices

Since the primary goal of the DH operator is to provide the heat demand in the network at lowest cost, we are looking at a cost-minimization problem. Therefore, the bidding to electricity market should be based on the principle that the participation in the electricity market is a means of cost reduction. We can apply a risk-averse bidding strategy by ensuring that we only submit bids that would lower our operational production costs in case they are accepted. Therefore, the bidding prices and amounts to the electricity market should be based on the heat production as shown in [2].

To determine the bidding prices based on the heat production setup at the DH plant, we consider what we call unit-switching prices. The unit-switching price is calculated between all heat-only units (\mathcal{U}^H) and market-dependent units ($\mathcal{U}^{CHP} \cup \mathcal{U}^{EL}$). The unit-switching price is the electricity price above (for CHP units) or below (electricity-driven units) which the heat production gets cheaper than with the respective heat-only unit (see [2] and [21] for CHP units). Figure 1 shows unit-switching on the example of a CHP unit, an electric boiler and a gas boiler for the Brønderslev system. Net heat production costs, meaning costs of producing heat minus power market revenues (in the case of a CHP unit) or power purchase costs (in the case of an electric boiler) are plotted on the y-axis with the corresponding electricity prices on the x-axis. The costs for the gas boiler stay flat whereas the curves for the CHP unit and electric boiler show a downward (upward) slope. The intersection points of these curves correspond to the unit-switching prices.

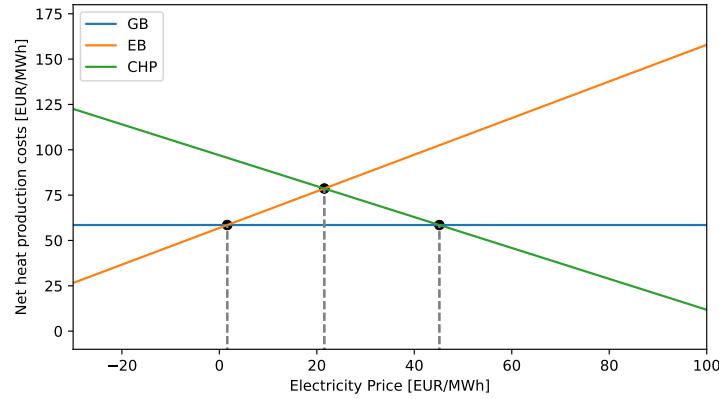


Fig. 1: Example of unit switching prices in Brønderslev system.

3.2 Generating sets of possible bids

If we assume that the market-dependent units always produce at full capacity due to efficiency reasons, the bidding amounts are fixed. This assumption can be made for the rather small units in DHS. Furthermore, since the prices can be calculated based on the operational heat production costs, the set of all possible bids to the electricity market can be generated. While generating the sets of possible bids, hourly bids but also block bids (grouping several hours together as one bid) can be considered. The procedure iterates through all market-dependent units and creates bids of all valid lengths, e.g., 1 hour or minimum three hours for block bids. The bidding amount is the capacity of the unit. Then for each unit-switching price of this unit with all heat-only units, a bid is generated. We refer to [21] for an extensive description of the bid generation. In case the unit has start-up costs, those are added to the bidding price by distributing them evenly across the bidding period.

3.3 Selection of bids to submit

Since not all possible bids can be submitted to the market due to capacity restrictions and a limited heat demand, we need to select a subset of bids. We can use the model presented in constraints (1) - (19) for the optimal selection of bids for a set of electricity price scenarios given in the set Ω . Let \mathcal{B} denote the set of all possible bids. The bids in \mathcal{B} can be distinguished in production bids \mathcal{B}^P for CHP units and consumption bids \mathcal{B}^C for electricity-consuming units. Then the value $v_{b,\omega}$ of bid b in scenario ω is defined as equation (20) or (21) depending on whether it is a producing or consuming unit, respectively, see [21] for the original definition.

$$v_{b,\omega} = \begin{cases} -a_b \sum_{t \in T_b} \lambda_{t,\omega}, & \text{if } p_b \leq \frac{\sum_{t \in T_b} \lambda_{t,\omega}}{|T_b|}, \\ 0, & \text{otherwise} \end{cases}, \quad \forall b \in \mathcal{B}^P, \omega \in \Omega \quad (20)$$

$$v_{b,\omega} = \begin{cases} a_b \sum_{t \in T_b} \lambda_{t,\omega}, & \text{if } p_b \geq \frac{\sum_{t \in T_b} \lambda_{t,\omega}}{|T_b|}, \\ 0, & \text{otherwise} \end{cases}, \quad \forall b \in \mathcal{B}^C, \omega \in \Omega \quad (21)$$

where a_b is the hourly bidding amount of the bid, T_b are the time periods of the bid and p_b is the bidding price (i.e. the unit-switching price). This means that the value of the bid is the income from (negative values) or price on the electricity market (positive values) in case the bid is accepted for production and consumption bids, respectively (i.e. the bidding price was above/below the average bidding price in the respective hours) and zero otherwise.

The following changes have to be made to the dispatch model: We add a set of binary variables $z_b \in \{0, 1\}$ that decide whether a bid b is selected ($z_b = 1$) or not ($z_b = 0$). z_b represents a first-stage decision in the stochastic program.

The income from the electricity markets in the objective function now depends on the bidding behaviour. Therefore, the original objective function (equation (1)) is changed to equation (22) using the value of the bid.

$$\text{Min} \sum_{\omega \in \Omega} \pi_{\omega} \sum_{t \in \mathcal{T}} \left[\sum_{u \in \mathcal{U}} C_u^H q_{u,t,\omega} + \sum_{u \in \mathcal{U}^C} C_u^S y_{u,t,\omega}^{\text{Start}} \right] + \sum_{b \in \mathcal{B}} \sum_{\omega \in \Omega} \pi_{\omega} v_{b,\omega} z_b \quad (22)$$

In case a bid b is selected, we have to ensure the bidding amount a_b is produced by the related unit u_b in the hours of the bid T_b . This is modelled by constraints (23). Finally, we have to exclude conflicting bids, i.e., bids that would require the same unit in the same hour. All conflicting bids to a bid b are collected in the set $\hat{\mathcal{B}}_b$ and can therefore be excluded in constraints (24).

$$a_b z_b = q_{u_b,t,\omega} \quad \forall b \in \mathcal{B}, t \in T_b, \omega \in \Omega \quad (23)$$

$$z_{b_1} + z_{b_2} \leq 1 \quad \forall b_1 \in \mathcal{B}, b_2 \in \hat{\mathcal{B}}_{b_1} \quad (24)$$

Thus, the full model for minimizing the expected operational cost including bid selection is described by equation system (25).

$$\begin{aligned} \text{Min} \quad & (22) \\ \text{s.t.} \quad & (2) - (19) \\ & (23) - (24) \end{aligned} \quad (25)$$

4 Demo Case

For the demo case in this chapter, we use data from a week in October 2020 in the Brønderslev DHS in a slightly modified version for readability. We optimize

the daily operation for one week (i.e. 168 hours) by running the model with a daily rolling horizon and planning period of 168 hours to account for storage behaviour.

The system contains 12 different units here given in descending order of operational cost (at a power price of 0 EUR/MWh): 7 small-scale CHP plants (CHP1-CHP7) each with a heat production capacity of 3.9 MW, two combined units of wood chip boilers with heat pumps (WCB+HP1 and WCB+HP2) each with a heat production capacity of 11 MW, one 35 MW natural gas-fired boiler (GB), one electric boiler (EB) producing up to 20 MW heat and one solar thermal unit (Solar). All of the units are connected to a storage unit with a capacity of 360 MWh. From the storage unit, three demand sites with at total of approx. 4800 customers can be supplied with heat.

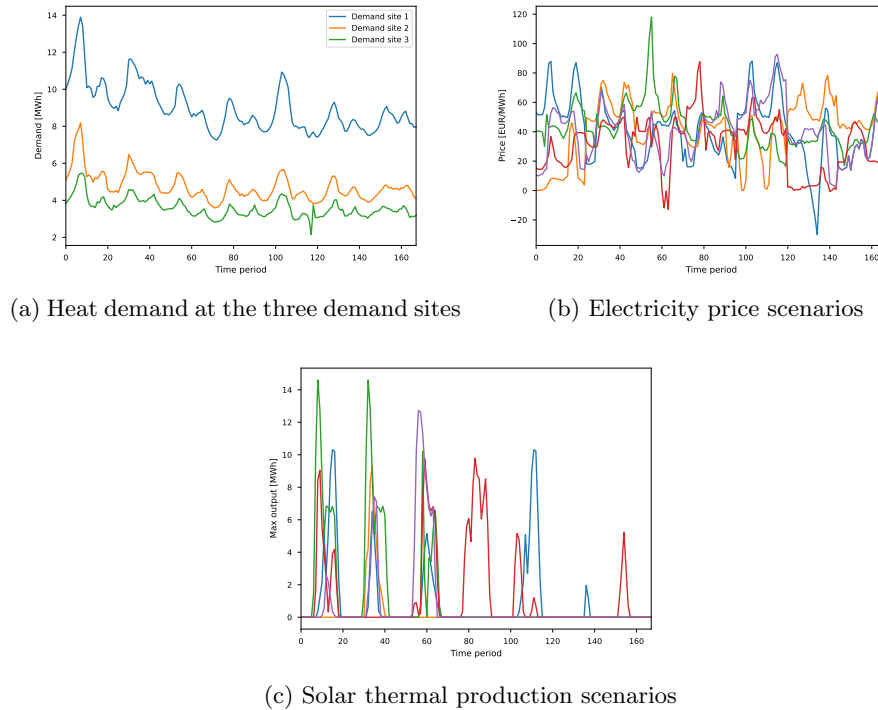


Fig. 2: Input data for the week in October 2020 (hourly scale)

The heat demand at the three demand sites assumed to be known and displayed in Figure 2a. The series follows a typical pattern with peaks in the morning and evening. We use five scenarios that contain the uncertain information about day-ahead market electricity prices and solar thermal production. The scenario data is depicted in Figures 2b and 2c, respectively. For the following

experiments, we assume that electricity can be sold on the day-ahead market using block bids with a minimum length of two hours. The heat pumps buy electricity at a fixed electricity price, while the electric boiler uses consumption bids to the day-ahead market.

4.1 Optimal dispatch without electricity market

Figure 3 shows the optimal production schedule for the real realization of the electricity prices and solar production when no income and buying from the electricity market is considered. To accomplish that, we have solved the stochastic program in Section 2 with five scenarios without the possibility of selling and buying on the day-ahead market. Afterwards, the dispatch of the units connected to the market is applied to the real prices and solar production. The heat production of the different heat generation units is displayed as an area chart.

We can see that heat generation from the gas boiler follows the demand curve most of the time. Solar heat supplies heat in the periods it is available. The mismatch between solar heat generation and heat demand is covered by heat storage.

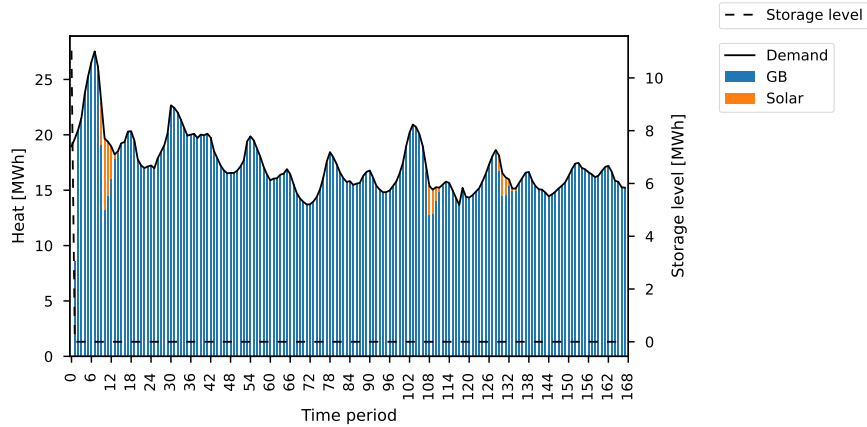


Fig. 3: Baseline dispatch without market participation for the week in October 2020

4.2 Optimal dispatch including block bidding.

In order to generate a set of possible bids to choose from (Sect. 3.1), unit-switching prices the CHP units and the other heat generation units have to be computed. The corresponding values are shown in Table 2.

Unit	Electric boiler	Gas boiler	WCB+HP	Solar
Switching price [EUR/MWh]	21.54	45.16	9.15	113.88

Table 2: Unit-switching prices between CHP units and the other heat producers in the Brønderslev system.

Figure 4 shows the heat dispatch when block bidding from CHP units and the electric boiler is included. In comparison to the baseline without block bidding (Fig. 3), mainly heat production from the gas boiler is replaced as power prices during the week of interest often exceed the unit-switching price of 45.16 EUR/MWh.

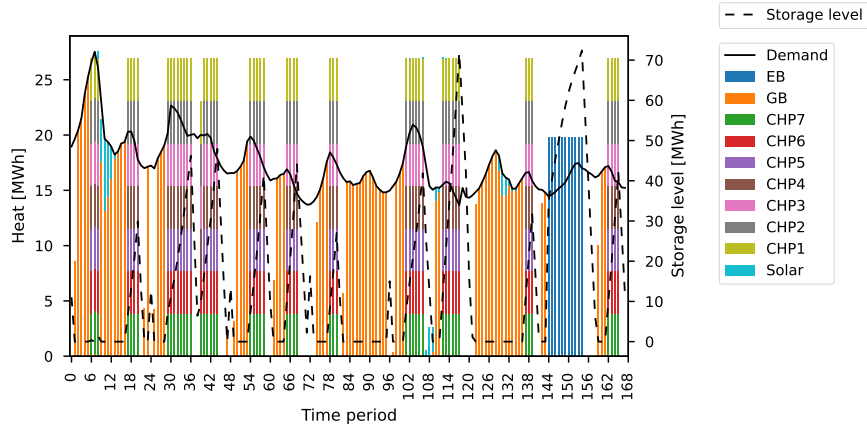


Fig. 4: Heat dispatch for the week in October 2020

The heat production corresponds to the co-generation of power (Fig. 5) from the seven CHP units in the hours where bids are won. Since power production is more economically viable during hours of high market prices, power production, and correspondingly heat production, does not follow heat demand. This mismatch between demand and supply is larger than previously (Fig. 3) and is compensated by the system's heat storage. Furthermore, the electric boiler consumes electricity some hours with low market prices (Fig. 6).

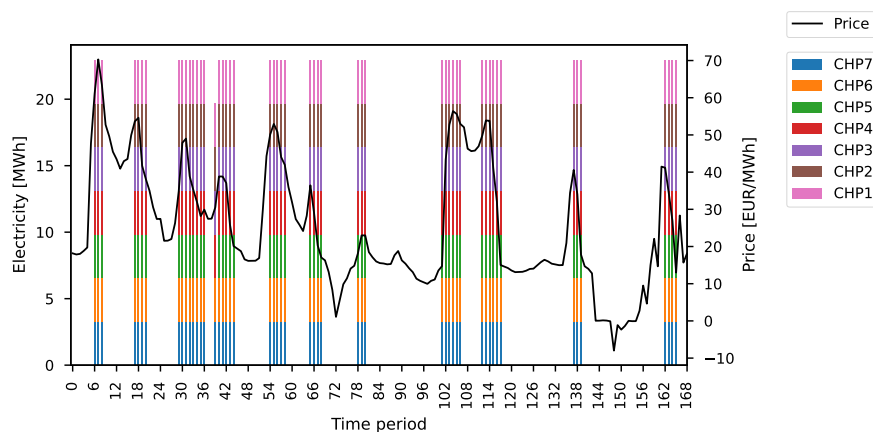


Fig. 5: Electricity production for the week in October 2020

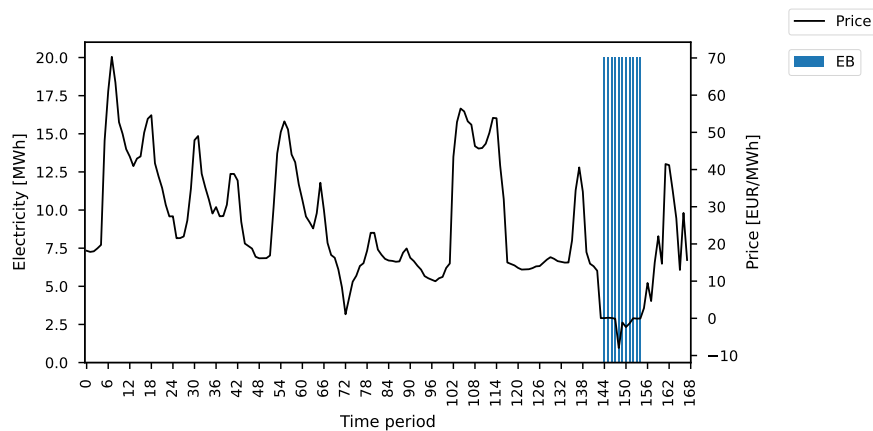


Fig. 6: Electricity consumption for the week in October 2020

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